R4457

Sub. Code
25MMT2C1

M.Sc. DEGREE EXAMINATION, APRIL – 2026

Second Semester

Mathematics

ADVANCED LINEAR ALGEBRA

(CBCS – 2025 onwards)

Time : 3 Hours Maximum : 75 Marks

Part A $(10 \times 1 = 10)$

Answer **all** questions by choosing the correct option.

1. Let V be a finite-dimensional vector space and $W \subseteq V$. Which of the following is true? (CO1, K2)

   (a) Every subset of V is a subspace

   (b) W is a subspace if it contains a nonzero vector

   (c) W is a subspace if it is closed under addition and scalar multiplication

   (d) W is a subspace if it is finite

2. Two matrices are row-equivalent if and only if they (CO1, K2)

   (a) Have the same determinant

   (b) Have the same rank

   (c) Represent the same linear transformation

   (d) Have identical row spaces

3. A linear transformation $T: V \to W$ is an isomorphism if and only if (CO2, K2)

(a) T is linear

(b) T is one-to-one

(c) T is onto

(d) T is linear, one-to-one and onto

4. A linear functional on a vector space V is (CO2, K2)

(a) A linear transformation from V to V

(b) A linear transformation from V to its dual

(c) A linear transformation from V to the field F

(d) A bilinear mapping on $V \times V$

5. The Lagrange interpolation theorem states that (CO3, K2)

(a) Every polynomial has a unique factorization

(b) A polynomial is uniquely determined by its roots

(c) Given distinct points, there exists a unique polynomial of least degree passing through them

(d) Every Polynomial is irreducible over a field

6. The determinant function is uniquely characterized by being (CO3, K2)

(a) Liner in each row only

(b) Alternating, multilinear and equal to 1 on the identity matrix

(c) Symmetric in the rows

(d) Invariant under all row operations

7. A vector space over a field F is a special case of (CO4, K2)

(a) A ring (b) An algebra

(c) A module over F (d) A group

8. A polynomial $p(x)$ is an annihilating polynomial for a linear operator T if. (CO4, K2)

(a) $p(T) = 1$

(b) $p(T) = 0$

(c) $p(x)$ divides the characteristic Polynomial

(d) $p(x)$ has distinct roots

9. A subspace W of V is invariant under a linear operator T if (CO5, K2)

(a) $T(W) \subseteq W$ (b) $T(W) = V$

(c) $W \subseteq \ker(T)$ (d) $W \cap T(W) = \{0\}$

10. A linear operator has Jordan canonical form if and only if (CO5, K2)

(a) It is diagonalizable

(b) Its characteristic polynomial splits over the field

(c) Its minimal polynomial has distinct roots

(d) It is invertible

Part B $(5 \times 5 = 25)$

Answer **all** questions not more than 500 words each.

11. (a) Prove that a non-empty subset W of V is a subspace of V if and only if for each pair of vectors α, β in W and each scalar c in F the vector $c\alpha + \beta$ is again in W. (CO1, K3)

Or

(b) Show that the subspace spanned by a non-empty subsets S of a vector space V is the set of all linear combinations of vectors in S. (CO1, K4)

12. (a) Let V be a finite-dimensional vector space over the field F and let $\{\alpha_1, \cdots, \alpha_n\}$ be an ordered basis for V. Let W be a vector space over the same held F and let $\{\beta_1, \cdots \beta_n\}$ be any vectors in W. Then prove that there is precisely one linear transformation T from V into W such that $T\alpha_j = \beta_j$, where $j = 1, \cdots, n$.

(CO2, K3)

Or

(b) Let V bean n-dimensional vector space over the field F and let W be an m-dimensional vector space over F. Then prove that the space $L(V, W)$ is finite-dimensional and has dimension mn. (CO2, K4)

13. (a) If f, d are polynomials over a field F and d is different from 0 then prove that there exist polyno mials q, r in $F[x]$ such that (CO3, K3)

(i) $f = dq + r$

(ii) either $r = 0$ or deg $r <$ deg d.

The polynomials q, r satisfying (i) and (ii) are unique.

Or

(b) If F is a field and M is any non-zero ideal in $F[x]$, show that there is a unique monic polynomial d in $F[x]$ such that M is the principal ideal generated by d. (CO3, K3)

14. (a) Let K be a commutative ring with identity. If V is a free K– module with n generators, then prove that the rank of V is n. (CO4, K3)

Or

(b) Let K be a field of characteristic zero and V a vector space over K. Show that the exterior product is an associative operation on the alternating multilinear forms on V. In other words, if L, M, and N are alternating multilinear forms on V of degrees r, s, and t, respectively, then prove that $(L \wedge M) \wedge N = L \wedge (M \wedge M)$. (CO4, K4)

15. (a) If W is an invariant subspace for T, then prove that W is invariant under every polynomial in T Thus, for each a in V, the conductor $S\ (\alpha; W)$ is an ideal in the polynomial algebra $F[x]$. (CO5, K4)

Or

(b) Let V be a finite-dimensional vector space over the field F and let T be a linear operator on V. Then prove that T is triangulable if and only if the minimal polynomial for T is a product of linear polynomials over F. (CO5, K4)

Part C $(5 \times 8 = 40)$

Answer **all** questions not more than 1000 words each.

16. (a) If W_1 and W_2 are finite-dimensional subspaces of a vector space V, then prove that $W_1 + W_2$ is finite-dimensional and *dim* $W_1 + \dim W_2 = \dim(W_1 \cap W_2) + \dim(W_1 + W_2)$. (CO1, K5)

Or

(b) Suppose P is an $n \times n$ it invertible matrix over F. Let V be an n-dimensional vector space over F, and let $\mathcal{B}$ be all ordered basis of V. Then show that there is a unique ordered basis $\mathcal{B}$ of V such that

(i) $[\alpha]_{\mathcal{B}} = P[\alpha]_{\mathcal{B}'}$

(ii) $[\alpha]_{\mathcal{B}'} = .P^{-1}[\alpha]_{\mathcal{B}}$ for every vecot α in V.

(CO1, K5)

17. (a) Let V be a finite-dimensional vector space over the field F and let W be a subspace of V. Then prove that $\dim W + \dim W^0 = \dim V$. (CO2, K4)

Or

(b) Let W_1 and W_2 be subspaces of a finite-dimensional vector space V. (CO2, K5)

(i) Prove that $(W_1 + W_2)^0 = W_1^0 \cap W_2^0$

(ii) Prove that $(W_1 \cap W_2)^0 = W_1^0 + W_2^0$

18. (a) State and prove Taylors Formula. (CO3, K5)

Or

(b) Let F be a field of characteristic zero and f a polynomial over F with deg $f \leq n$ Then prove that the scalar c is a root of f of multiplicity r if and only if $(D^k f)(c) = 0,\ 0 \leq k \leq r-1$ or $(D^k f)(c) \neq 0$.

(CO3, K5)

19. (a) Let K be a commutative ring with identity and let V be a module over K. Then show that the exterior product is an associative operation on the alternating multilinear forms on V. In other words, if L, M, and N are alternating multilinear forms on V of degrees r, s, and t, respectively, then $(L \wedge M) \wedge N = L \wedge (M \wedge N)$. (CO4, K6)

Or

(b) State and prove Cayley-Hamilton theorem.

(CO4, K4)

20. (a) Let T be a linear operator on the finite-dimensional vector space V over the field F. Let p be the minimal polynomial for $T, p = p_1^{r_1}, \cdots, p_k^{r_k}$ where the p_i are distinct irreducible monic polynomials over F and the r_1 are positive integers. Let W_i be the null space of $p_i(T)^{r_j}$, $i = 1, \cdots k$, The prove that (CO5, K4)

(i) $V = W_1 \oplus \cdots \oplus W_k$;

(ii) each W_i is invariant under T;

(iii) if T_i is the operator induced on W_i by T, then prove that the minimal polynomial for T_i is $p_i^{r_1}$.

Or

(b) Let T be a linear operator on the finite-dimensional vector space V over the field F. Suppose that the minimal polynomial for T decomposes over F into a product of linear polynomials. Then prove that there is a diagonalizable operator D on V and a nilpotent operator Non V such that (CO5, K4)

(i) $T = D + N$

(ii) DN = ND

The diagonalizable operator D and the nilpotent operator N are uniquely determined by (i) and (ii) and each of them is a polynomial in T.

R4458

Sub. Code
25MMT2C2

M.Sc. DEGREE EXAMINATION, APRIL – 2026

Second Semester

Mathematics

REAL ANALYSIS – II

(CBCS – 2025 onwards)

Time : 3 Hours Maximum : 75 Marks

Part A $(10 \times 1 = 10)$

Answer **all** the following objective type questions by choosing the correct option.

1. The unit step function $I(x)$ is defined as (CO1, K1)

   (a) $I(x)=1$ for $x \le 0$, $I(x)=0$ for $x>0$

   (b) $I(x)=x$ for $x>0$, $I(x)=-x$ for $x \le 0$

   (c) $I(x)=0$ for $x \le 0$, $I(x)=1$ for $x>0$

   (d) $I(x)=0$ for all x

2. If a curve y is one-to-one, it is called (CO1, K1)

   (a) Closed curve

   (b) Arc

   (c) Interval

   (d) Mapping

3. Let $f_n(x) = \dfrac{1}{nx+1}$, $0 < x < 1, n = 1,2,3,....$ Which of the following statements is correct? (CO2, K2)

(a) $f_n(x) \to 0$ pointwise and monotonically, but not uniformly on (0, 1)

(b) $f_n(x) \to 0$ uniformly on (0, 1)

(c) $\{f_n(x)\}$ does not converge on (0, 1)

(d) $f_n(x) \to 1$ uniformly on (0, 1)

4. If X is a metric space, what does $C_b(X)$ denote? (CO2, K1)

(a) The set of all complex-valued functions on X

(b) The set of all continuous functions on X

(c) The set of all bounded functions on X

(d) The set of all complex-valued, continuous, bounded functions on X

5. Which of the following statements correctly describes the Weierstrass Approximation Theorem? (CO3, K2)

(a) Every polynomial can be approximated uniformly by continuous functions on $[a,b]$

(b) Every continuous function on $[a,b]$ can be uniformly approximated by polynomials

(c) Only differentiable functions on $[a,b]$ can be approximated by polynomials

(d) Polynomials cannot approximate continuous functions on $[a,b]$

6. Let $\mathcal{A}$ be a self-adjoint algebra of complex-valued continuous functions on a compact set $\mathcal{K}$. If $\mathcal{A}$ separates points of $\mathcal{K}$ and vanishes at no point of $\mathcal{K}$, then which statement is true? (CO2, K1)

(a) $\mathcal{A}$ consists only of constant functions

(b) The uniform closure of $\mathcal{A}$ is a proper subset of $C(\mathcal{K})$

(c) $\mathcal{A}$ is dense in $C(\mathcal{K})$

(d) $\mathcal{A}$ is not closed under complex conjugation

7. Let $f(x) = x^3 - \sin^2 x \tan x$, $g(x) = 2x^2 - \sin^2 x - x \tan x$ for $x \in \left(0, \frac{\pi}{2}\right)$. Determine the sign of each function. (CO4, K2)

(a) $f(x) > 0$ for all $x, g(x) > 0$ for all x

(b) $f(x) < 0$ for all $x, g(x) < 0$ for all x

(c) $f(x)$ changes sign, $g(x) < 0$ for all x

(d) Both $f(x)$ and $g(x)$ change sign

8. The function $\phi_n(x) = \frac{1}{\sqrt{2\pi}} e^{inx}$, $n \in \mathbb{Z}$ on the interval $[-\pi, \pi]$ are (CO4, K1)

(a) Orthonormal

(b) Orthogonal but not normalized

(c) Linearly dependent

(d) None of the above

9. Evaluate $\lim_{x \to 0} (1+x)^{\frac{1}{x}}$ (CO5, K2)

(a) 0 (b) 1

(c) e (d) ∞

10. Evaluate $\lim_{x \to 0} \frac{x - \sin x}{\tan x - x}$ (CO5, K2)

(a) 0 (b) $\frac{1}{2}$

(c) 1 (d) $\frac{2}{3}$

Part B ($5 \times 5 = 25$)

Answer **all** questions not more than 500 words each.

11. (a) State and prove the change of variables. (CO1, K3)

Or

(b) Prove that $f \in \mathscr{R}(\alpha)$ on $[a,b]$ if and only if for every $\in > 0$ these exists a partition P such that $U(P,f,\alpha) - L(P,f,\alpha) < \in$. (CO1, K4)

12. (a) State and prove the Cauchy Criterion for uniform convergence. (CO2, K3)

Or

(b) Prove that every uniformly convergent sequence of bounded functions is uniformly bounded. (CO2, K4)

13. (a) If f_n is a pointwise bounded sequence of complex functions on a countable set E, the prove that f_n has a subsequence f_{nk} such that $f_{nk}(x)$ converge for every $x \in E$. (CO3, K4)

Or

(b) Let $\mathscr{B}$ be the uniform closure of an algebra $\mathscr{A}$ of bounded functions. Then prove $\mathscr{B}$ is a uniformly closed algebra. (CO3, K5)

14. (a) Suppose $f(x) = \sum_{n=0}^{\infty} C_n \chi^n$, the series converging in $|x| < \mathbb{R}$. If $\mathbb{R} < a < \mathbb{R}$, then prove the f can be expanded in a power series about the point $x = a$ which converges in $|x - a| < \mathbb{R} - |a|$, and

$$f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)(a)}}{n!}(x-a)^n \left(|x-a| < \mathbb{R} - |a|\right).$$ (CO4, K5)

Or

(b) Suppose the series $\sum a_n \chi^n$ and $\sum b_n \chi^n$ converge in the segment $S = (-\mathbb{R}, \mathbb{R})$. Let E be the set of all $x \in S$ at which $\sum_{n=0}^{\infty} a_n \chi^n = \sum_{n=0}^{\infty} b_n \chi^n$. If E has a limit point in S, then prove $a_n = b_n$ for $n = 0,1,2,...$ Hence $\sum_{n=0}^{\infty} a_n \chi^n = \sum_{n=0}^{\infty} b_n \chi^n$ holds for $x \in S$. (CO4, K4)

15. (a) If, for some x, there are constants $\delta > 0$ and $M < \infty$ such that $|f(x+t) - f(x)| \le M|t|$ for all $t \in (-\delta, \delta)$, then prove $\lim_{N \to \infty} S_N(f; x) = f(x)$. (CO5, K4)

Or

(b) If $x > 0$ and $y > 0$, then prove that

$$\int_0^1 t^{x-t}(1-t)^{y-t}\,dt = \frac{\Gamma(x)\Gamma(y)}{\Gamma(x+y)}.$$ (CO5, K4)

Part C (5 × 8 = 40)

Answer **all** questions not more than 1000 words each.

16. (a) Suppose f is bounded on $[a,b]$, f has only finitely many points of discontinuity on $[a,b]$, and α is continuous at which f is discontinuous. Then prove $f \in \mathscr{R}(\alpha)$. (CO1, K4)

Or

(b) Assume α increases monotonically and $\alpha' \in \mathscr{R}$ on $[a,b]$. Let f be a bounded real function on $[a,b]$. Then show that $f \in \mathscr{R}(\alpha)$ is and only if $f\alpha' \in \mathscr{R}$. In that case $\int_a^b f d\alpha = \int_a^b f(x)\alpha'(x)dx$. (CO1, K5)

17. (a) Let f_n be a sequence of continuous function which converges uniformly to a function f on a set E. Prove that $\lim_{n\to\infty} f_n(x_n) = f(x)$ for very sequence of points $x_n \in E$ such that $x_n \to x$, and $x \in E$, Is the converse of this true? (CO2, K5)

Or

(b) Show that $(C(X), \|\cdot\|)$ into a complete metric space.

(CO2, K5)

18. (a) Show that there exists a real continuous function on the real line which is nowhere differentiable.

(CO3, K4)

Or

(b) State and prove the Stone-Weierstrass theorem.

(CO3, K6)

19. (a) Let e^x be defined on $\mathbb{R}'$ by $E(x) = e^x$ and $E(z) = \sum_{n=0}^{\infty} \frac{Z^n}{n!}$. Then prove the following (CO4, K4)

(i) e^x is continuous and differentiable for all x;

(ii) $(e^x)' = e^x$;

(iii) e^x is a strictly increasing function of x, and $e^x > 0$;

(iv) $e^{x+y} = e^x e^y$.

(v) $e^x \to +\infty$ as $x \to +\infty$, $e^x \to 0$ as $x \to -\infty$;

(vi) $\lim_{x \to +\infty} \chi^n e^{-x} = 0$, for every n.

Or

(b) (i) Let Φ_m be orthonormal on $[a,b]$. Let $S_n(x) = \sum_{m=1}^{n} C_m \phi_m(x)$ be the nth partial sum of the Fourier series of f, and suppose $t_n(x) = \sum_{m=1}^{n} \gamma_m \phi_m(x)$. Then prove $\int_a^b |f - S_n|^2 dx \leq \int_a^b |f - t_n|^2 dx$, and equality holds if and only if $\gamma_m = C_m$ (m = 1, 2, ...,n).

(ii) If $f(x) = 0$ for all x in some segment J, then prove $\lim S_N(f; x) = 0$ for every $x \in J$.

(CO4, K4)

20. (a) Put $f(x) = x$ if $0 \le x \le 2\pi$, and apply Parseval's theorem to conclude that $\sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{\pi^2}{6}$. (CO5, K5)

Or

(b) If f is a positive function on $(0, \infty)$ such that

(i) $f(x+1) = xf(x)$,

(ii) $f(1) = 1$,

(iii) $\log f$ is convex,

then prove $f(x) = \Gamma(x)$. (CO5, K4)

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R4459

Sub. Code
25MMT2C3

M.Sc. DEGREE EXAMINATION, APRIL – 2026

Second Semester

Mathematics

COMPLEX ANALYSIS

(CBCS – 2025 onwards)

Time : 3 Hours Maximum : 75 Marks

Part A $(10 \times 1 = 10)$

Answer **all** the following objective type questions by choosing the correct option.

1. If two harmonic function u and v satisfy the Cauchy-Riemann equations in a domain then (CO1, K1)

   (a) $u + iv$ is continuous but not analytic

   (b) $u + iv$ is analytic in the domain

   (c) $u = v$ everywhere in the domain

   (d) u and v are constant

2. Using the series expansion, which of the following is the correct Maclaurin series of $\cos z$? (CO1, K1)

   (a) $1-\frac{z^2}{2!}+\frac{z^4}{4!}-\cdots$

   (b) $z-\frac{z^3}{3!}+\frac{z^5}{5!}-\cdots$

   (c) $1+\frac{z^2}{2!}+\frac{z^4}{4!}-\cdots$

   (d) $\frac{z^2}{2!}-\frac{z^4}{4!}-\cdots$.

3. Let γ be a closed curve lying entirely inside a circle $|z| = \mathbb{R}$. For a point a such that $|a| > \mathbb{R}$ the index $n(\gamma, a)$ is (CO2, K2)

(a) 1 (b) –1

(c) 0 (d) $2\pi i$

4. In Cauchy's Theorem for a disk, the region Ω in which the function is assumed to be is analytic (CO2, K1)

(a) a closed disk

(b) an annulus

(c) an open disk $|z - z_0| < \rho$

(d) the entire complex plane

5. The Maximum Modulus Principle implies that for a nonconstant analytic function $f(z)$ in a region Ω, then (CO3, K1)

(a) $|f(z)|$ can have both a maximum and a minimum inside Ω

(b) $|f(z)|$ has no local maximum inside Ω

(c) $|f(z)|$ is bounded below by zero only on the boundary

(d) $f(z)$ must have a zero inside Ω

6. If an analytic function assumes values arbitrarily close to every complex number in every neighbourhood of a point z_0, then z_0 is (CO3, K1)

(a) Removable singularity

(b) Pole

(c) Essential singularity

(d) Regular point

7. Residue of the function $\frac{1}{\sin z}$ at the singular points is (CO4, K2)

(a) -1 (b) 1

(c) -2 (d) $(-1)^n$

8. Which of the following functions is harmonic in the region $r > 0$? (CO4, K1)

(a) r^2 (b) $\log r$

(c) e^r (d) $r\cos\theta$

9. In the Laurent expansion of $f(z) = \frac{1}{(z-1)(z-2)}$ valid in the region $1 < |z| < 2$, the coefficient of $\frac{1}{z^2}$ is (CO5, K2)

(a) 0 (b) $\frac{1}{2}$

(c) 1 (d) -1

10. The principal branch of arcsin z has real part in the interval (CO5, K2)

(a) $(-\pi, \pi)$ (b) $(-\pi/2, \pi/2)$

(c) $(0, \pi)$ (d) $(-\infty, \infty)$

Part B $(5 \times 5 = 25)$

Answer **all** questions not more than 500 words each.

11. (a) If all zeros of a polynomial $P(z)$ lie in a half plane. then prove that all zeros of the derivative $p'(z)$ lie in the same half plane. (CO1, K2)

Or

(b) Prove that the functions $u(z)$ and $u(\overline{z})$ are simultaneously harmonic. (CO1, K3)

12. (a) State and explain Cauchy's integral formula for an analytic function $f(z)$ in a domain D, include the condition on the point a and the curve γ. (CO2, K5)

Or

(b) Assume that $f(z)$ is analytic and satisfies the inequality $|f(z)-1|<1$ in a region Ω. Show that $\int_{\gamma} \frac{f'(z)}{f(z)} dz = 0$. (CO2, K3)

13. (a) Prove that an analytic function comes arbitrarily close to any complex value in every neighbourhood of an essential singularity. (CO3, K3)

Or

(b) If $f(z)$ is defined and continuous on a closed bounded set E and analytic on the interior of E, then prove that the maximum of $|f(z)|$ on E is assumed on the boundary of E. (CO3, K2)

14. (a) Evaluate the following integrals by the method of residues. (CO4, K5)

(i) $\int_{-\infty}^{\infty} \frac{x^2 - x + 2}{x^4 + 10x^2 + 9} dx$

(ii) $\int_{0}^{\infty} \frac{\cos x}{(x^2 + a^2)} dx$ a real.

Or

(b) State and prove the Mean-value Property for harmonic functions. Deduce the maximum principle for harmonic function. (CO4, K4)

15. (a) If the functions $f_n(z)$ are analytic and $\neq 0$ in a region Ω, and if $f_n(z)$ converges to $f(z)$, uniformly on every compact subset of Ω. then prove $f(z)$ is either identically zero or never equal to zero in Ω. (CO5, K2)

Or

(b) Using Taylor's theorem applied to a branch of $\log(1+z/n$, prove that $\lim_{n\to}\left(1+\frac{z}{n}\right)^n = e^z$. (CO5, K6)

Part C (5 × 8 = 40)

Answer **all** questions not more than 1000 words each.

16. (a) (i) Prove that the cross ratio (z_1, z_2, z_3, z_4) is real if and if the four points lie on a circle or on a straight line.

(ii) If $g(w)$ and $f(z)$ are analytic functions, show that $g(f(z))$ is also analytic. (CO1, K5)

Or

(b) State and prove Abel's limit theorem. (CO1, K6)

17. (a) State and prove the necessary and sufficient condition for the line integral $\int_\gamma (pdx + qdy)$ to be independent of the path in a domain Ω. (CO2, K4)

Or

(b) Compute $\int_{|z|=p} \frac{|dz|}{|z-a|^2}$ under the condition $|a| \neq \rho$ by using of the equations $z\bar{z} = \rho^2$ and $|dz| = -i\rho \frac{dz}{z}$.

(CO2, K5)

18. (a) State and prove Cauchy's Theorem for an analytic function defined on a region Ω. (CO3, K6)

Or

(b) Suppose that $f(z)$ is analytic in the region Ω' obtained by omitting a point a from a region Ω. Prove that a necessary and sufficient condition that there exist an analytic function in Ω which coincides with $f(z)$ in Ω' is that $\lim_{z\to a}(z-a)f(z) = 0$ and extended function is uniquely determined.

(CO3, K3)

19. (a) State Rouche's theorem and illustrate it with an application to determine the number of zeros of an analytic function inside a given disk. (CO4, K6)

Or

(b) Suppose that $f(z)$ is analytic in the annulus $r_1 < |z| < r_2$ and continuous on the closed annulus. If $M(r)$ denotes the maximum of $|f(z)|$ for $|z| = r$, show that

$$M(r) \leqq M(r_1)^{\alpha} M(r_2)^{1-\alpha}$$

where $\alpha = \log(r_2/r) : \log(r_2/r_1)$ (Hadamard's three-circle theorem). Discuss cases of equality by applying the maximum principle to a linear combination of $\log|f(z)|$ and $\log|z|$.

(CO4, K5)

20. (a) Suppose that $f_n(z)$ is analytic in the region Ω_n and that the sequence $\{f_n(z)\}$ converges to a limit function $f(z)$ in a region Ω, uniformly on every compact subset of Ω. Then prove that $f(z)$ is analytic in Ω. Moreover, $f_n'(z)$ converges uniformly to $f'(z)$ on every compact subset of Ω. (CO5, K6)

Or

(b) If $f(z)$ is analytic in the region Ω, containing z_0, then show that the representation

$$f(z) = f(z_0) + \frac{f'(z_0)}{1!}(z - z_0) + \cdots + \frac{f^{(n)}(z_0)}{n!}(z - z_0)^n$$ is

valid in the largest open disk of center z_0 contained in Ω (CO5, K5)

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R4460

Sub. Code
25MMT2C4

M.Sc. DEGREE EXAMINATION, APRIL – 2026

Second Semester

Mathematics

ADVANCED PARTIAL DIFFERENTIAL EQUATIONS

(CBCS – 2025 onwards)

Time : 3 Hours Maximum : 75 Marks

Part A ($10 \times 1 = 10$)

Answer **all** the following objective type questions by choosing the correct option.

1. A curve in three dimensions defined by $x=f(t),\ y=g(t), z=h(t)$ is called (CO1, K1)

   (a) An implicit curve

   (b) A parametric curve

   (c) A level surface

   (d) A Pfaffian curve

2. The gradient vector ∇F at a point on the surface $F(x, y, z) = 0$ is (CO1, K2)

   (a) Tangent to the surface

   (b) Parallel to the surface

   (c) Normal to the surface

   (d) Lying on the surface

3. In the equation $z = ax + by + ab$, eliminating the constants a and b gives (CO2, K2)

(a) $z = xy$

(b) $z^2 = 4xy$

(c) $z = 0$

(d) $x + y = z$

4. Which of the following is a first-order but not first-degree partial differential equation? (CO2, K1)

(a) $p + q = z$ (b) $xp + yq = z$

(c) $p^2 + q = x$ (d) $p + q = x$

5. Jacobi's method is mainly used to find (CO3, K2)

(a) Singular integrals

(b) Complete integrals

(c) General solutions

(d) Particular solutions

6. Charpit's method reduces a nonlinear first-order PDE into (CO3, K1)

(a) A system of ordinary differential equations

(b) A linear partial differential equation

(c) A second-order partial differential equation

(d) An algebraic equation

7. Which of the following equations is elliptic? (CO4, K1)

(a) $z_{xx} - z_{yy} = 0$ (b) $z_{xx} + z_{yy} = 0$

(c) $z_{xx} - 2z_{xy} + z_{yy} = 0$ (d) $z_{xx} = 0$

8. Which of the following is a second-order but first-degree partial differential equation? (CO4, K2)

(a) $z_{xx}^2 + z_{yy} = 0$ (b) $z_{xx} + z_{yy} + z_x = 0$

(c) $(z_{xx})^3 + z = 0$ (d) $\sqrt{z_{xx}} + z_y = 0$

9. The Laplace equation is classified as (CO5, K1)

(a) Hyperbolic (b) Parabolic

(c) Elliptic (d) Degenerate

10. Which equation models steady-state heat conduction? (CO5, K1)

(a) Wave equation

(b) Diffusion equation

(c) Laplace equation

(d) Poisson equation

Part B (5 × 5 = 25)

Answer **all** questions not more than 500 words each.

11. (a) Find the integral curves of the equations

$$\frac{dx}{y(x+y)+az} = \frac{dy}{x(x+y)-az} = \frac{dz}{z(x+y)}.$$ (CO1, K3)

Or

(b) Solve the equations $\frac{dx}{y+az} = \frac{dy}{z+\beta x} = \frac{dz}{x+\gamma y}$ (CO1, K4)

12. (a) Eliminate the arbitrary Function f from the equations $z = xy + f(x^2 + y^2)$ and $z = x + y + f(xy)$. (CO2, K3)

Or

(b) Find the general solution of the differential equation $x^2 \frac{\partial z}{\partial x} + y^2 \frac{\partial z}{\partial y} = (x+y)z$. (CO2, K4)

13. (a) Show that the equations $xp = yq, z(xp + yq) = 2xy$ are compatible and solve them. (CO3, K3)

Or

(b) Find the complete integral of the equation $z^2 = pqxy$. (CO3, K4)

14. (a) If $u = f(x + iy) + g(x - iy)$, where the functions f and g are arbitrary, then show that $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$. (CO4, K3)

Or

(b) If u is the complementary function and z_1 a particular integral of a linear partial differential equation, then prove that $u + z_1$ is a general solution of the equation. (CO4, K3)

15. (a) If $\rho > 0$ and $\psi(r) = \int_V \frac{\rho(r')\,d\tau'}{|r - r'|'}$ where the volume V is bounded, then prove that $\lim_{r=\infty} r\psi(r) = \int_V \rho(r')\,d\tau'$. (CO5, K3)

Or

(b) Derive d'Alembert's solution of the one-dimensional wave equation. (CO5, K4)

Part C (5 × 8 = 40)

Answer **all** questions not more than 1000 words each.

16. (a) Find the orthogonal trajectories on the surface $x^2+y^2+2fyz+d=0$ of its curves of intersection with planes parallel to the plane xOy. (CO1, K3)

Or

(b) Find the integral curves of the equations $\frac{dx}{x+z}=\frac{dy}{y}=\frac{dz}{z+y^2}$. (CO1, K4)

17. (a) If $u_i(x_1,x_2,\ldots,x_n,z)=c_i\,(i=1,2,\ldots,n)$ are independent solutions of the equations $\frac{dx_1}{p_1}=\frac{dx_2}{p_2}=\ldots=\frac{dx_n}{p_n}=\frac{dz}{R}$ then prove that the relation $\Phi(u_1, u_2,..u_n)=0$, in which the function Φ is arbitrary, is a general solution of the linear partial differential equation $p_1\frac{\partial z}{\partial x_1}+p_2\frac{\partial z}{\partial x_2}+\ldots+p_n\frac{\partial z}{\partial x_n}=R$. (CO2, K3)

Or

(b) Find the solution of the equation $z=\frac{1}{2}\left(p^2+q^2\right)+(p-x)(q-y)$ which passes through x-axis. (CO2, K4)

18. (a) Show that the equation $xpq+yq^2=1$ has complete integrals

(i) $(z+b)^2=4(ax+y)$

(ii) $kx(z+h)=k^2y+x^2$ and deduce (ii) from (i). (CO3, K3)

Or

(b) Find the complete integral of the equation $zpq=p+q$. (CO3, K4)

19. (a) Find the solution of the equation $\frac{\partial^2 z}{\partial x^2} - \frac{\partial^2 z}{\partial y^2} = x - y$.

(CO4, K3)

Or

(b) Determine the solution of the equation $\frac{\partial^4 z}{\partial x^4} + \frac{\partial^2 z}{\partial y^2} = 0$ satisfying the conditions

(i) z and its partial derivatives tend to zero as $x \to \pm\infty$

(ii) $z = f(x), \frac{\partial z}{\partial y} = 0$ on $y = 0$. (CO4, K3)

20. (a) The points of trisection of a string are pulled aside through a distance $\in$ on opposite sides of the position of equilibrium, and the string is released from rest. Derive an expression for the displacement of the string at any subsequent time and show that the mid-point of the string always remains at rest.

(CO5, K3)

Or

(b) Determine the temperature $\theta(\rho, t)$ in the infinite cylinder $0 \le \rho \le a$ when the initial temperature is $\theta(\rho, 0) = f(\rho)$ and the surface $\rho = a$ is maintained at zero temperature. (CO5, K4)

R4461

Sub. Code
25MMT2E3

M.Sc. DEGREE EXAMINATION, APRIL – 2026

Second Semester

Mathematics

Elective – PYTHON PROGRAMMING

(CBCS – 2025 onwards)

Time : 3 Hours Maximum : 75 Marks

Part A $(10 \times 1 = 10)$

Answer **all** the following objective type questions by choosing the correct option.

1. ________________ feature makes Python easy to read and write. (CO1, K1)

   (a) Compilation (b) Indentation

   (c) Linking (d) Encryption

2. ________________ data type is used to store decimal values in Python. (CO1, K1)

   (a) int (b) str

   (c) float (d) bool

3. ________________ library is mainly used for numerical computations in Python. (CO2, K1)

   (a) Pandas (b) NumPy

   (c) SymPy (d) TensorFlow

4. ________________ SciPy module is used for optimization problems. (CO2, K2)

   (a) scipy.signal (b) scipy.stats

   (c) scipy.optimize (d) scipy.linalg

5. _______________ Pandas structure represents one-dimensional labeled data. (CO3, K2)

(a) DataFrame (b) Array

(c) Series (d) Matrix

6. _______________ Matplotlib plot is suitable for showing data distribution. (CO3, K1)

(a) Line plot (b) Bar chart

(c) Histogram (d) Scatter plot

7. _______________ SymPy function is used for differentiation. (CO4, K2)

(a) integrate( ) (b) diff( )

(c) limit( ) (d) solve( )

8. _______________ operation is used to solve equations symbolically. (CO4, K2)

(a) diff( ) (b) integrate( )

(c) solve( ) (d) simplify( )

9. _______________ type of learning uses labeled data. (CO5, K1)

(a) Unsupervised learning

(b) Reinforcement learning

(c) Supervised learning

(d) Deep learning

10. _______________ library is commonly used for machine learning in Python. (CO5, K1)

(a) NumPy (b) Pandas

(c) Scikit-learn (d) Matplotlib

Part B (5 × 5 = 25)

Answer **all** questions not more than 500 words each.

11. (a) Illustrate the features and basic syntax of Python. (CO1, K3)

Or

(b) Describe Python data types in detail with examples. (CO1, K2)

12. (a) Explain NumPy arrays and basic matrix operations. (CO2, K3)

Or

(b) Describe the role of SciPy in scientific computing. (CO2, K4)

13. (a) Illustrate Data Frame in detail with example. (CO3, K3)

Or

(b) Describe in detail about preprocessing techniques. (CO3, K2)

14. (a) Explain symbolic differentiation and integration using SymPy. (CO4, K4)

Or

(b) Describe the procedure to solve equations using SymPy. (CO4, K4)

15. (a) What is supervised learning? Explain in detail. (CO5, K4)

Or

(b) How to implement machine learning model using scikit-learn? Explain. (CO5, K5)

Part C (5 × 8 = 40)

Answer **all** questions not more than 1000 words each.

16. (a) Explain in detail about control structures in Python. (CO1, K2)

Or

(b) Write Python program to find factorial and check prime number. (CO1, K6)

17. (a) Explain in detail about NumPy mathematical function with examples. (CO2, K4)

Or

(b) Discuss SciPy applications in optimization and signal processing. (CO2, K2)

18. (a) Explain in detail data manipulation and visualization using Pandas. (CO3, K3)

Or

(b) Illustrate line, bar, scatter and histogram plots using Matplotlib. (CO3, K3)

19. (a) Explain symbolic mathematics concepts using SymPy with examples. (CO4, K4)

Or

(b) Discuss matrix operations using SymPy. (CO4, K2)

20. (a) Explain in detail about basics of tensor flow for deep learning. (CO5, K4)

Or

(b) Write Python program to find the greatest of three numbers and factorial of a number. (CO5, K6)

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R4462

Sub. Code
25MMT2S1

M.Sc. DEGREE EXAMINATION, APRIL – 2026

Second Semester

Mathematics

LATEX

(CBCS – 2025 onwards)

Time : 3 Hours Maximum : 75 Marks

Part A ($10 \times 1 = 10$)

Answer **all** the questions by choosing the correct option.

1. LaTeX was created by (CO1, K1)

   (a) Alexia Gaudeul

   (b) Edward

   (c) Leslie Lamport

   (d) Donald E.Knuth

2. What is the purpose of the command \usepackage{graphicx} in LaTeX? (CO1, K1)

   (a) It sets the page margins

   (b) It includes graphics into the document

   (c) It adds a table of contents

   (d) It sets the font size

3. Which of the following is a valid way to write a fraction in math mode? (CO2, K1)

(a) \fraction{a} {b}

(b) a/b

(c) \frac{a} {b}

(d) \div a b

4. Which symbol is produced by the command \in?(CO2, K2)

(a) $\subset$

(b) $\in$

(c) $\forall$

(d) $\parallel$

5. Which command produces bold text? (CO3, K1)

(a) \textit { }

(b) \emph{ }

(c) \textbf{ }

(d) \underline{ }

6. Which command is used to produce a summation symbol with limits? (CO3, K2)

(a) \sum_{i=1}^n

(b) \Sigma_{i=l}^n

(c) \add_{i=l}^n

(d) \plus_{i=l}^n

7. The expression code for $\frac{x^2+1}{x-1}$ in LaTeX is (CO4, K1)

(a) (\frac{x^2+1}{x-1})

(b) \big(\frac{x^2+1}{x-1}\big)

(c) \left(\frac{x^2+1}{x-1}\right)

(d) \frac{(x^2+1)}{(x-1)}

8. Which is correct syntax for Latex? (CO4, K2)

(a) \begin{ }...\stop{ }

(b) \begin{ }...\end{ }

(c) \start{ }...\stop{ }

(d) \start{ }...\end{ }

9. Output of $\frac{x^{2}}{2x_{ 1 }}\times y$ is equal to (CO5, K1)

(a) $\frac{x^2}{2x_1}y$

(b) $\frac{x^3}{x_1}y_1$

(c) $\frac{x_1^2}{2x}y$

(d) $\frac{y^2}{x_1}x$

10. The command \subseteq produces (CO5, K1)

(a) $\subset$

(b) $\subseteq$

(c) $\in$

(d) $\supseteq$

Part B (5 × 5 = 25)

Answer **all** questions not more than 500 words each.

11. (a) Explain about plain tex. (CO1, K2)

Or

(b) Discuss tex program. (CO1, K3)

12. (a) Discuss page numbering in LaTeX (CO2, K2)

Or

(b) Derive and explain the procedure used to format text in multiple columns. (CO2, K3)

13. (a) Discuss one-sided justification and two-sided indentation. (CO3, K3)

Or

(b) Discuss choice of font size. (CO3, K2)

14. (a) Explain LR boxes creation. (CO4, K2)

Or

(b) Discuss the standard footnotes. (CO4, K2)

15. (a) How do we define function names in LaTeX? Give an example. (CO5, K3)

Or

(b) What are the mathematical accents available in math mode? (CO5, K2)

Part C $(5 \times 8 = 40)$

Answer **all** questions not more than 1000 words each.

16. (a) Explain environment in detail. (CO1, K4)

Or

(b) Enumerate the special characters in LaTeX and discuss their roles in document preparation. (CO1, K3)

17. (a) Describe the essential features of version 2.3 from 2000/06/28. (CO2, K3)

Or

(b) Explain page format in LaTeX. (CO2, K4)

18. (a) Discuss nested lists and their properties. (CO2, K4)

Or

(b) List out style parameters. (CO3, K3)

19. (a) Discuss construction of tables in LaTeX. (CO4, K4)

Or

(b) Explain parboxes and minipages in LaTeX. (CO4, K3)

20. (a) Discuss the concept of automatic sizing of bracket symbols in LaTeX, highlighting the commands used to scale delimiters appropriately for complex mathematical expressions. (CO5, K4)

Or

(b) List the commonly used relation symbols in LaTeX along with their corresponding negation symbols, and discuss their usage briefly with suitable explanations. (CO5, K4)

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R4948

Sub. Code
511401

M.Sc. DEGREE EXAMINATION, APRIL – 2026

Fourth Semester

Mathematics

FUNCTIONAL ANALYSIS

(CBCS – 2022 onwards)

Time : 3 Hours Maximum : 75 Marks

Section A $(10 \times 1 = 10)$

Answer **all** questions by choosing the correct option.

1. A Banach space is a ______________ (CO1, K1)

   (a) Complete normed space

   (b) Normed space

   (c) Hilbert space

   (d) Inner product space

2. Which of the following statement is correct? (CO1, K2)

   (a) The dual space of l^1 is l^1

   (b) The dual space of l^1 is l^2

   (c) The dual space of l^1 is l^∞

   (d) The dual space of l^1 is c_0

3. An element x of an inner product space X is said to be orthogonal to an element $y \in X$ if ______________ (CO2, K2)

(a) $\langle x, y \rangle \neq 0$ (b) $\langle y, x \rangle = 0$

(c) $\langle y, x \rangle \neq 0$ (d) $\langle x, y \rangle = 0$

4. An orthonormal set in an inner product space is always ______________ in X. (CO2, K1)

(a) linearly dependent

(b) a basis of the space

(c) a spanning set

(d) linearly independent

5. The Hilbert adjoint operator T^* of T is defined by ______________ (CO3, K1)

(a) $\langle Tx, y \rangle = \langle x, T^* y \rangle$

(b) $\langle Tx, y \rangle = \langle x, Ty \rangle$

(c) $\langle Tx, x \rangle = \langle x, T^* y \rangle$

(d) $\langle T^* x, y \rangle = \langle x, T^* y \rangle$

6. A real unitary matrix is ______________ (CO3, K1)

(a) a symmetric matrix

(b) a diagonal matrix

(c) an orthogonal matrix

(d) singular matrix

7. Every vector space $X \neq \{0\}$ has a ______________ (CO3, K1)

(a) Lower bound (b) Hamel basis

(c) Orthonormal set (d) Lattice

8. If T is represented by a matrix T_E, then the adjoint operator $T^{\times}$ is represented by __________ (CO4, K1)

(a) T_E

(b) T_E^{-1}

(c) the transpose of T_E

(d) the determinant of T_E

9. Let X and Y be normed spaces. A sequence (T_n) of operators $T_n \in B(X, Y)$ is said to be __________ if (T_n) converges in the norm on $B(X, Y)$. (CO5, K1)

(a) Convergent

(b) Uniformly operator convergent

(c) Strongly operator convergent

(d) Weakly operator convergent

10. An algebra A is called a division algebra if __________ (CO5, K1)

(a) every element of A is invertible

(b) every nonzero element of A is invertible

(c) A has no identity element

(d) only scalar elements of A are invertible

Section B (5 × 5 = 25)

Answer **all** questions not more than 500 words each.

11. (a) Prove that every finite-dimensional subspace Y of a normed linear space X is complete and hence, deduce that every finite-dimensional normed space is complete. (CO1, K4)

Or

(b) Let $T : \mathscr{D}(T) \to Y$ be a linear operator, where $\mathscr{D}(T) \subset X$ and X, Y are normed spaces. Then Prove that : (CO1, K5)

(i) T is continuous if and only if T is bounded.

(ii) If T is continuous at a single point, it is continuous.

12. (a) Show that the space $C[a, b]$ is not an inner product space, hence not a Hilbert space. (CO2, K5)

Or

(b) If Y is a closed subspace of a Hilbert space H. Then prove that $Y = Y^{\perp\perp}$. (CO2, K4)

13. (a) Let X and Y be inner product spaces and $Q : X \to Y$ a bounded linear operator. Then prove that

(i) $Q = 0$ if and only if $< Qx, y >= 0$ for all $x \in X$ and $y \in Y$.

(ii) If $Q : X \to X$, where X is complex, and $< Qx, x >= 0$ for all $x \in X$, then $Q = 0$. (CO3, K4)

Or

(b) Prove that the product of two bounded self-adjoint linear operators S and T on a Hilbert space H is self-adjoint if and only if the operators commute, $ST = TS$. (CO3, K5)

14. (a) Prove that in every Hilbert space $H \neq \{0\}$ there exists a total orthonormal set. (CO4, K5)

Or

(b) Prove that Every Hilbert space H is reflexive. (CO4, K5)

15. (a) Let (x_n) be a sequence in a normed space X. Then prove that

(i) Strong convergence implies weak convergence with the same limit.

(ii) The converse of (a) is not generally true.

(iii) If dim $X < \infty$, then weak convergence implies strong convergence. (CO5, K5)

Or

(b) Let $T : \mathscr{D}(T) \to Y$ be a bounded linear operator with domain $\mathscr{D}(T) \subset X$, where X and Y are normed spaces. Prove that : (CO5, K5)

(i) If $\mathscr{D}(T)$ is a closed subset of X, then T is closed.

(ii) If T is closed and Y is complete, then $\mathscr{D}(T)$ is a closed subset of X.

Section C (5 × 8 = 40)

Answer **all** questions not more than 1000 words each.

16. (a) State and prove Riesz's lemma. (CO1, K5)

Or

(b) Prove that the dual space of l^p is l^q, where $1 < p < \infty$ and q is the conjugate of p, that is, $\frac{1}{p} + \frac{1}{q} = 1$. (CO1, K5)

17. (a) Let X be an inner product space and let $M \neq \emptyset$ be a convex subset of X which is complete with respect to the metric induced by the inner product. Then prove that for every $x \in X$, there exists a unique $y \in M$ such that

$$\delta = \inf_{\bar{y} \in M} \|x - \bar{y}\| = \|x - y\|.$$ (CO2, K5)

Or

(b) Prove : Two Hilbert spaces H and $\tilde{H}$, both real or both complex, are isomorphic if and only if they have the same Hilbert dimension. (CO2, K5)

18. (a) Let T be a bounded linear operator on a Hilbert space H. Show that the Hilbert-adjoint operator T^* of T exists, is unique, and is itself a bounded linear operator. Further, prove that $\|T^*\| = \|T\|$.

(CO3, K5)

Or

(b) Let $T : H \rightarrow H$ be a bounded linear operator on a Hilbert space H. Then prove that

(i) If T is self-adjoint, $< Tx, x >$ is real for all $x \in H$.

(ii) If H is complex and $< Tx, x >$ is real for all $x \in H$, the operator T is self-adjoint. (CO3, K5)

19. (a) Prove that the adjoint operator $T^{\times}$ is linear and bounded, and $\|T^{\times}\| = \|T\|$. (CO4, K5)

Or

(b) Prove that if the dual space X' of a normed space X is separable, then X is separable. (CO4, K5)

20. (a) State and prove Closed Graph Theorem. (CO5, K4)

Or

(b) Prove that all matrices representing a given linear operator $T: X \to X$ on a finite-dimensional normed space X, relative to various bases for X, have the same eigenvalues. (CO5, K5)

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R4949

Sub. Code
511402

M.Sc. DEGREE EXAMINATION, APRIL – 2026

Fourth Semester

Mathematics

PROBABILITY AND STATISTICS

(CBCS – 2022 onwards)

Time : 3 Hours Maximum : 75 Marks

Part A $(10 \times 1 = 10)$

Answer **all** questions by choosing the correct option.

1. $P(A|B) =$ (CO1, K1)

   (a) $P(A \cap B)/P(B)$ (b) $P(A \cap B)/P(A)$

   (c) $P(A \cup B)/P(B)$ (d) $P(A \cup B)/P(A)$

2. Suppose X is a real-valued random variable with $E[X] = 4$ and $E[X^2] = 25$. Which of the following statements is true? (CO1, K1)

   (a) $E[X^2] > (E[X])^2$

   (b) $E[X^2] = (E[X])^2$

   (c) $E[X^2] < (E[X])^2$

   (d) It is impossible to determine without more information

3. ____________ is the subset of sample space (CO2, K1)

(a) event

(b) random scale

(c) outcomes

(d) random experiment

4. Identify the incorrect statement regarding the expectation of random variables: (CO2, K1)

(a) $E(X+Y)=E(X)+E(Y)$

(b) $E(aX+Y)=a\,E(X)+E(Y)$

(c) $E(XY)=E(X).E(Y)$

(d) $E(X^2)=(E(X))^2$

5. Consider a Poisson distribution for the tossing of a biased coin. The mean for this distribution is μ. The standard deviation for this distribution is given by (CO3, K1)

(a) $\sqrt{\mu}$ (b) μ^2

(c) μ (d) $1/\mu$

6. Consider a set of 18 samples from a standard normal distribution. We square each sample and sum all the squares. The number of degrees of freedom for a Chi Square distribution will be (CO3, K1)

(a) 18 (b) 17

(c) 20 (d) 19

7. ____________ is the branch of mathematics for collecting, analysing and interpreting data. (CO4, K1)

(a) probability (b) random variable

(c) statics (d) statistic

8. What will be the 't value' when 'between-groups variance' and 'within-groups variance' is 100 and 50 respectively (CO4, K1)

(a) 4 (b) $\sqrt{2}$

(c) 2 (d) 8

9. ____________ is a rule for calculating an estimate of a given quantity based on observed data. (CO5, K1)

(a) equivocator (b) approximate

(c) estimator (d) none of the above

10. Convergence in probability is ____________ convergence in distribution. (CO5, K1)

(a) equal to (b) weaker than

(c) stronger than (d) equal and stronger than

Part B $(5 \times 5 = 25)$

Answer **all** questions not more than 500 words each.

11. (a) Derive the law of total probability. (CO1, K2)

Or

(b) Let X be a random variable with cumulative distribution function $F(x)$. Prove that (CO1, K2)

(i) For all a and B, if $a < b$, then $F(a) \le F(b)$ (F is non-decreasing).

(ii) $\lim_{x \to -\infty} F(x) = 0$

(iii) $\lim_{x \to \infty} F(x) = 1$

(iv) $\lim_{x \downarrow x_0} F(x) = F(x_0)$

12. (a) Let X_1 and X_2 have the pdf $f(x_1, x_2) = \begin{cases} 8x_1 x_2 & 0 < x_1 < x_2 < 1 \\ 0 & \text{otherwise} \end{cases}$. Find $E(X_1 X_2^2)$ and $E(7X_1 X_2^2 + 5X_2)$. (CO2, K2)

Or

(b) Let (X_1, X_2) be a random vector. Let $Y_1 = g_1(X_1, X_2)$ and $Y_2 = g_2(X_1, X_2)$ be random variables whose expectations exist. Then show that for all real numbers k_1 and k_2 ,

$E(k_1 Y_1 + k_2 Y_2) = k_1 E(Y_1) + k_2 E(Y_2)$. (CO2, K2)

13. (a) Let X be a random variable such that $E(X^m) = \frac{(m+3)!}{3!} 3^m$, $m = 1, 2, 3, \ldots$ Then find the mgf of X. (CO3, K3)

Or

(b) Suppose X has a $\chi^2(r)$ distribution. If $k > -r/2$, show that $E(X^k)$ exists and $E(X^k) = \frac{2^k \Gamma\left(\frac{r}{2} + k\right)}{\Gamma\left(\frac{r}{2}\right)}$

if $k > -r/2$. (CO3, K3)

14. (a) Find the mean and variance of the t-distribution. (CO4, K2)

Or

(b) Let Y_1, Y_2, Y_3 be the order statistics of a random sample of size 3 from a distribution having pdf

$$f(x)=\begin{cases}1 & 0<x<1\\ 0 & \text{elsewhere}\end{cases}$$

Calculate the pdf of the sample range $Z_1 = Y_3 - Y_1$. (CO4, K3)

15. (a) Suppose $X_n \xrightarrow{P} a$ and the real function g is continuous at a. Then show that $g(X_n) \xrightarrow{P} g(a)$. (CO5, K3)

Or

(b) Let T_n, have a t-distribution with n degrees of freedom, $n=1,2,3,...$ and its cdf is

$$F_n(t)=\int_{-\infty}^{t} \frac{\Gamma[(n+1)/2]}{\sqrt{n n}\,\Gamma(n/2)} \frac{1}{(1+y^2/n)^{(n+1)/2}} dy,$$

where the integrand is the pdf $f_n(y)$ of T_n. Show that T_n has a limiting standard normal distribution. (CO5, K3)

Part C (5 × 8 = 40)

Answer **all** questions not more than 1000 words each.

16. (a) State and prove Chebychev's inequality. (CO1, K4)

Or

(b) (i) State and Prove Boole's Inequality.

(ii) Show that for any random variable, $P[X=x] = F_X(x) - F_X(x-), \forall x \in R$, where $F_x(x-) = \lim_{z\uparrow x} F_X(z)$. (CO1, K3)

17. (a) Suppose the joint mgf, $M(t_1, t_2)$, exists for the random variables X_1 and X_2. Then show that X_1 and X_2 are independent if and only if the joint mgf is identically equal to the product of the marginal mgfs. (CO2, K2)

Or

(b) (i) Let X_1 and X_2 have the joint pdf

$$f(x_1, x_2) = \begin{cases} x_1 + x_2 & 0 < x_1 < 1, 0 < x_2 < 1 \\ 0 & \text{elsewhere} \end{cases}$$

find the probability $P\left(X_1 \leq \frac{1}{2}\right)$ and $P(X_1 + X_2 < 1)$. (CO2, K2)

(ii) Prove that the random variables X_1 and X_2 are independent random variables if and only if the following conditions holds,

$P(a < X_1 \leq b, c < x_2 \leq d) =$

$P(a < X_1 \leq b, c < X_2 \leq d)$ for every $a < b$ and $c < d$, where a, b, c, d are constants.

18. (a) (i) If the random variable X is $N(\mu, \sigma^2)$, $\sigma^2 > 0$, then show that the random variable $V = \frac{(X-\mu)^2}{\sigma^2}$ is $\chi^2(1)$.

(ii) Prove the additive property of the normal distribution under independence. (CO3, K3)

Or

(b) Let $X_1,\ldots X_n$ be independent random variables. Suppose, for $i=1,\ldots n$, that X_i has a $\Gamma(\alpha_i,\beta)$ distribution. Let $Y=\sum_{i=1}^{n} X_i$. Then prove that Y has distribution. $\Gamma\left(\sum_{i=1}^{n} \alpha_i,\beta\right)$ distribution. (CO3, K3)

19. (a) Let $Y_1<Y_2<\ldots<Y_n$ denote the n order statistics based on the random sample $X_1, X_2\ldots,X_n$, from a continuous distribution with pdf $f(x)$ and support (a,b). Then derive the joint pdf of $Y_1,Y_2,\ldots Y_n$. (CO4, K4)

Or

(b) Explain F-distribution. (CO4, K4)

20. (a) State and prove the weak law of large numbers. (CO5, K3)

Or

(b) State and prove central limit theorem. (CO5, K4)

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R4950

Sub. Code
511403

M.Sc. DEGREE EXAMINATION, APRIL – 2026

Fourth Semester

Mathematics

GRAPH THEORY

(CBCS – 2022 onwards)

Time : 3 Hours Maximum : 75 Marks

Part A $(10 \times 1 = 10)$

Answer **all** the following objective type questions by choosing the correct option.

1. The incidence matrix of a graph is defined as (CO1, K1)
   (a) Matrix showing adjacency of vertices
   (b) Matrix showing adjacency of edges
   (c) Matrix showing vertex-edge incidences
   (d) None of the above

2. In a tree with n vertices, the number of edges is (CO1, K1)
   (a) $n-1$ (b) n
   (c) $n+1$ (d) $2n-1$

3. A Hamiltonian cycle is (CO2, K2)
   (a) A cycle passing through every vertex exactly once
   (b) A cycle passing through every edge exactly once
   (c) A path passing through all vertices
   (d) A subgraph containing a cycle

4. The connector problem in graph theory is associated with (CO2, K1)

   (a) Finding minimum spanning trees

   (b) Finding Eulerian circuits

   (c) Finding Hamiltonian paths

   (d) Matching vertices

5. Turán's theorem provides an upper bound on the (CO3, K1)

   (a) Number of edges in a graph

   (b) Number of vertices in a graph

   (c) Number of independent sets

   (d) Size of the largest clique-free graph

6. The size of a maximum independent set in a graph is called its (CO3, K1)

   (a) Matching number

   (b) Independence number

   (c) Vertex cover number

   (d) Chromatic number

7. Brooks' theorem states that the chromatic number of a graph is at most (CO4, K1)

   (a) $\Delta(G)+1$ (b) $\Delta(G)$

   (c) $\Delta(G)-1$ (d) $\Delta(G)+2$

8. Hajós' conjecture is related to (CO4, K1)

(a) Chromatic polynomials

(b) Coloring planar graphs

(c) Coloring non-planar graphs

(d) Complete graphs

9. The four-color conjecture states that (CO5, K1)

(a) Four colors suffice to color any planar graph

(b) Any graph can be colored with four colors

(c) Planar graphs require at least four colors

(d) None of the above

10. The dual graph of a planar graph is obtained by. (CO5, K1)

(a) Connecting adjacent regions with edges

(b) Connecting adjacent vertices with edges

(c) Connecting opposite regions with edges

(d) None of the above

Part B $(5 \times 5 = 25)$

Answer **all** questions not more than 500 words each.

11. (a) Show that any two longest paths in a connected graph have a vertex in common. (CO1, K3)

Or

(b) Prove that a vertex v of a tree G is a cut vertex of G if and only if $d(v) > 1$. (CO1, K4)

12. (a) If G is hamiltonian then prove that for every nonempty proper subset S of V $\omega(G-S) \leq |S|$.

(CO2, K2)

Or

(b) Prove that $c(G)$ is well defined. (CO2, K5)

13. (a) Let G be a bipartite graph with bipartition (X, Y). Then prove that G contains a matching that saturates every vertex in X if and only if $|N(S)| \geq |S|$ for all $S \subseteq X$. (CO3, K4)

Or

(b) Prove that in a bipartite graph, the number of edges in a maximum matching is equal to the number of vertices in a minimum covering. (CO3, K3)

14. (a) Let G be a connected graph that is not an odd cycle. Then prove that G has a 2-edge colouring in which both colours are represented at each vertex of degree at least two. (CO4, K5)

Or

(b) In a critical graph. Prove that no vertex cut is a clique. (CO4, K5)

15. (a) Let v be a vertex of a planar graph G. Then prove that G can be embedded in the plane in such a way that v is on the exterior face of the embedding.

(CO5, K3)

Or

(b) Prove K_5 is nonplanar. (CO5, K4)

Part C $(5 \times 8 = 40)$

Answer **all** the questions not more than 1000 words each.

16. (a) Prove that $\tau(K_n) = n^{n-2}$. (CO1, K4)

Or

(b) Show that if an edge e is in a closed trial of G, then prove that e is in a cycle of G. (CO1, K6)

17. (a) Let G be a simple graph with degree sequence $(d_1, d_2, \cdots d_v)$, where $d_1 \leq d_2 \leq \cdots \leq d_v$ and $v \geq 3$ Suppose that there is no value of m less than $v/2$ for which $d_m \leq m$ and $d_{v-m} \leq v-m$. Then show that G is hamiltonian. (CO2, K5)

Or

(b) Show that the graph $H_{m,n}$ is m-connected.

(CO2, K3)

18. (a) Show that G has a perfect matching if and only if $o(G-S) \leq |S|$ for all $S \subset V$. (CO3, K4)

Or

(b) Show that a matching M in G is a maximum matching if and only if G contains no M-augmenting path. (CO3, K2)

19. (a) Prove that if G is simple then either $\chi' = \Delta$ or $\chi' = \Delta + 1$. (CO4, K6)

Or

(b) If G is a connected simple graph and is neither an odd cycle nor a complete graph, then prove that $\chi \leq \delta$. (CO4, K5)

20. (a) Prove that every planar graph is 5-vertex-colourable. (CO5, K4)

Or

(b) Show that a plane graph G is 2-face colourable if and only if G is eulerian. (CO5, K4)

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R4951

Sub. Code
511404

M.Sc. DEGREE EXAMINATION, APRIL – 2026

Fourth Semester

Mathematics

MEASURE AND INTEGRATION

(CBCS – 2022 onwards)

Time : 3 Hours Maximum : 75 Marks

Part A ($10 \times 1 = 10$)

Answer **all** questions by choosing the correct option.

1. $m^*(\phi) =$ __________. (CO1, K2)

   (a) 0 (b) 1

   (c) ∞ (d) $-\infty$

2. Ess sup $f =$ __________. (CO1, K2)

   (a) –ess inf $(-f)$

   (b) ess inf $(-f)$

   (c) –ess inf (f)

   (d) ess inf (f)

3. If $f = 0$ a.e., then __________. (CO2, K2)

   (a) $\int f dx = 0$ (b) $\int f dx = \infty$

   (c) $\int f dx = 1$ (d) none

4. $\int_0^1 \left(\frac{\log x}{1-x}\right)^2 dx =$ (CO2, K3)

(a) 1 (b) $\frac{\pi^3}{3}$

(c) 0 (d) ∞

5. Let f be integrable with indefinite integral F, then __________. (CO3, K3)

(a) $F' = f$ a.e (b) $F' \neq f$ a.e

(c) $F' = f$ not a.e (d) none

6. $\lim_{h \to 0} \frac{1}{h} \int_0^h |f(x+t) - f(x)| dt =$ __________. (CO3, K2)

(a) 1 (b) 0

(c) ∞ (d) $-\infty$

7. A countable union of sets negative with respect to a __________ ν is a negative (CO4, K2)

(a) signed measure (b) measure R

(c) null set (d) all the above

8. If μ is a measure, $\int f d\mu$ exists and $\nu = \int_E d\mu$ then __________. (CO4, K3)

(a) $\nu << \mu$ (b) $\nu >> \mu$

(c) $\nu = \mu = 0$ (d) all the above

9. If $f(x, y) = \dfrac{x^2 - y^2}{(x^2 + y^2)^2}, (x, y) \neq (0,0)$, then $\int_0^1 dx \int_0^1 f(x, y) dy =$ __________ and $\int_0^1 dy \int_0^1 f(x, y) dx =$ __________ (CO5, K2)

(a) $\infty, -\infty$

(b) $-\dfrac{\pi}{2}, \dfrac{\pi}{2}$

(c) $\dfrac{\pi}{4}, -\dfrac{\pi}{4}$

(d) 0, 0

10. $(S \times J) =$ __________. (CO5, K2)

(a) 1

(b) $M_0(\xi)$

(c) 0

(d) none

Part B (5 × 5 = 25)

Answer **all** the questions not more than 500 words each.

11. (a) Prove that for any sequence of sets $E_i, m^*\left(\bigcup_{i=1}^{\infty} E_i\right) \leq \sum_{i=1}^{\infty} m^*(E_i)$. (CO1, K5)

Or

(b) Prove that every measurable set is not a Borel set. (CO1, K4)

12. (a) State and prove the Fatou's lemma. (CO2, K3)

Or

(b) Show that $\int_0^1 \frac{x^{\frac{1}{3}}}{1-x} \log \frac{1}{x}\, dx = 9\sum_{n=1}^{\infty} \frac{1}{(3n+1)^2}$.

(CO2, K5)

13. (a) Prove that if $f \in BV[a,b]$, then $f(b) - f(a) = P - N$ and $T = P + N$, all variations being on the finite interval $[a,b]$. (CO3, K4)

Or

(b) Prove that if $f \in L(a,b)$ and $\int_a^x f dt = 0$ for all $x \in (a,b)$ then $f = 0$ almost everywhere in (a,b).

(CO3, K3)

14. (a) Show that if $\phi(E) = \int_E f d\mu$ where $\int f d\mu$ is defined, then ϕ is a signed measure. (CO4, K3)

Or

(b) State and prove the Hahn decompositions theorem.

(CO4, K5)

15. (a) Prove that ξ is an algebra. (CO5, K4)

Or

(b) State and prove the Fubini's theorem. (CO5, K6)

Part C (5 × 8 = 40)

Answer **all** the questions not more than 1000 words each.

16. (a) (i) Prove that the outer measure of an interval equals its length.

(ii) Prove that every interval is measurable. (CO1, K2)

Or

(b) Show that if $m^*(E) < \infty$ then E is measurable if and only if, $\forall \in > 0$, $\exists$ disjoint finite intervals $I_1,\ldots I_n$ such that $m^*\left(E \Delta \bigcup_{i=1}^{n} I_i\right) < \in$. We may stipulate that the intervals I_i be open, closed or half-open. (CO1, K2)

17. (a) State and prove Lebesgue's Dominated Convergence Theorem. (CO2, K4)

Or

(b) Prove that if f be a bounded function defined on the finite interval $[a,b]$, then f is Riemann integrable over $[a,b]$ if and only if it is continuous almost everywhere. (CO2, K4)

18. (a) State and prove the Lebesgue's differentiation theorem. (CO3, K6)

Or

(b) Prove that if $f \in L(a,b)$ where (a,b) is a finite interval, then there exists a set $E \subseteq (a,b)$ such that $m([a,b]-E) = 0$ and $\lim_{h \to 0} \frac{1}{h} \int_x^{x+h} |f(t) - \xi| dt = |f(x) - \xi|$ for all real ξ and all $x \in E$. (CO3, K6)

19. (a) State and prove the Random-Nikodyn theorem. (CO4, K3)

Or

(b) State and prove the Jordan decomposition theorem. (CO4, K3)

20. (a) Prove that $\mathcal{A}$ is an algebra, $S(\mathcal{A}) = \mathcal{M}(\mathcal{A})$, that is, the σ algebra generated by $\mathcal{A}$ is the smallest monotone class containing $\mathcal{A}$. (CO5, K5)

Or

(b) Prove that if $[[X, \mathcal{S}, \mu]]$ and $[[Y, \mathcal{J}, \nu]]$, v be $\sigma -$finite measure spaces. For $V \in \mathcal{S} \times \mathcal{J}$ write $\phi(x) = \nu(V_x), \psi(y) = \mu(V^y)$, for each $x \in X, y \in Y$. Then ϕ is S-measurable, ψ is J-measurable and $\int_X \phi du = \int_Y \psi d\nu$. (CO5, K5)